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Computation of the Nucleolus of Some Bilateral Market Games[1]

By P. Legros[2]

Abstract: This paper gives a simple algorithm for computing the nucleolus of bilateral markets with two complementary commodities.

Introduction

Maschler, Peleg and Shapley give a geometric characterization of the central place of the nucleolus in the imputation space. Their method is related to the algorithm of Kopelowitcz which consists of solving a sequence of linear programs. Papers of Brune, Owen, Kohlberg, Bruynell and Grotte give some general results concerning the properties and the computation of the nucleolus. However, little is known about the evolution of the nucleolus within a given class of games; exceptions are results of Littlechild for special cost games, of Galil for weighted majority games, and of Driessen and Tijs for special economic games. The purpose of this paper is to compute the nucleolus of the following class of games.

We consider the class of cooperative games with side-payments and whose characteristic function is,

$$v(S) = |S \cap P| \wedge \lambda \cdot |S \cap Q| \quad \text{all } S \subset P \cup Q \tag{1.1}$$

where $P \cup Q$ is the set of players
$\wedge$ is the min operator: $a \wedge b = \min(a, b)$
$\lambda \in \mathbb{R}_+^*$.

This is typically a generalization of the well known gloves game (take $\lambda = 1$). We can consider for instance that P is a set of manufacturers, each of whom owns a machine, and that Q is a set of workers each willing to work a certain amount of time. Then $v(S)$ represents the net profit that the coalition S can get with the output produced by

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[2] Patrick Legros, Université de Paris XII (La Varenne St. Hilaire). Current address: California Institute of Technology, Humanities, Pasadena, CA 91125, USA.

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