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# 'ESSENTIAL' PATENTS, FRAND ROYALTIES AND TECHNOLOGICAL STANDARDS*

Mathias Dewatripont†
Patrick Legros‡

Standard Setting Organizations have developed FRAND agreements in order to prevent firms from holding up other participants once a standard is created. We analyze here the consequences of such agreements—in particular the requirements of fairness and non-discrimination—for the creation of technological standards that require the participation of existing patent holders. We abandon the usual assumption that patents bring known benefits to the industry or that their benefits are known to all parties. When royalty payments are increasing in one's patent portfolio, as is implicitly the case in FRAND agreements, private information about the quality of patents leads to a variety of distortions, in particular the incentives of firms to 'pad' by contributing patents that are 'inessential' for the given standard, a phenomenon that seems to be widespread. Several results emerge from the analysis: (i) the number of inessential patents co-varies positively with the number of essential patents; (ii) there is over-investment relative to the second-best, that is when padding cannot be avoided and (iii) the threat of disputes reduces incentives to pad but at the cost of lower production of strong patents; (iv) mitigating this undesirable side-effect calls for a simultaneous increase in the cost of padding, through a better filtering of patent applications.

## I. INTRODUCTION

While firms try to differentiate themselves from competitors, they also benefit from having standards established: this facilitates in particular the access to other providers' consumers and allows economies of scale in the production of various inputs (e.g., chipsets, other electronic parts, etc). Coordination problems make reliance on market mechanisms ill-fitted for standards creation, especially when the standards to be developed are

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†Authors' affiliations: Université libre de Bruxelles, ECARES and CEPR, 50 Ave. F. D. Roosevelt, 1050 Brussels, Belgium.
‡Université libre de Bruxelles, ECARES and CEPR, 50 Ave. F. D. Roosevelt, 1050 Brussels, Belgium.

complex.[1] For this reason, different industries have established standards setting organizations (SSO) whose primary role is to facilitate coordination between firms in the industry and other stakeholders.

Before the 1980's, many of these organizations did not deal with technologies (for instance, standards for voltage increments in light bulbs) that embodied significant intellectual property rights. However, since the 1980's, in parallel to the rise in ICT, the need for interoperability led to standards that embodied significant IPR; firms started to claim a 'fair' return on previous investment in R&D that generated the patents that a standard could potentially infringe upon.

In telecommunications only, at least seven SSO's exist.[2] Each of these SSO's deals with many different standard processes (see Chiao *et al*. [2007], or Rysmann and Simcoe [2008]) each involving hundreds of participants and thousands of patents. Because the creation of a standard often involves the combination of technologies that are complementary, antitrust authorities tend to have a permissive approach toward these cooperative efforts (see for instance Schmalensee [2009]).

One dimension of uncertainty faced by the participants and contributors to the SSO is the level of royalties that will eventually be charged by the patent holders once the standard is established. These royalties depend on the contribution of each firm to the standard and also on the 'essentiality' of their patents, that is, the possibility of using the standard without infringing on these patents. What firms may anticipate however is that patent owners may opportunistically charge high royalties since, once a standard emerges and is adopted, it is costly for a single firm to produce a good which differs from the standard. This problem is particularly important when the creation of a standard requires the use of many different innovations, which is typically the case in high technology industries (in the case of mobile telephony for instance, a handset can require the use of more than one thousand technologies protected by patents).

Opportunistic behavior has two effects. The first is well documented in the literature and is related to its static effects: the anticipation of opportunistic behavior may discourage participation in SSO's. This has led SSO's to design rules of conduct to limit the possibilities of hold-up, in order to

[1] See Farrell and Saloner [1988] on the tradeoff between cooperative and market based mechanisms for standardization. They consider a world where the technologies are already available and the question is whether one or two standards will eventually survive. In this paper, and in the reality of markets, standard creation is done in parallel with the development of technologies. Bolton and Farrell [1990] analyze the costs of delay in adoption and duplication costs when solutions to problems are decentralized rather than centralized.

[2] Namely, Alliance for Telecommunications Industry Solutions (ATIS), European Telecommunications Industry Solutions (ATIS), European Telecommunications Standards Institute (ETSI), Internet Engineering Task Force (IETF), International Telecommunications Union (ITU), Open Mobile Alliance (OMA) and Telecommunications Industry Association (TIA).

increase the willingness to participate in the development of a standard. FRAND is a leading example of such rules (or RAND in the U.S., e.g., Swanson and Baumol [2005]). In a FRAND agreement, firms agree to contribute to the standard all the patents that are 'essential' to this standard and to settle on royalties that are 'fair, reasonable and non-discriminatory.'[3] The ND part means that all downstream firms have to be treated 'equally;' the FR part, although admittedly a bit vague, means that royalty rates should not be 'excessive.' Note however that such a requirement does not prevent firms with more patents from asking for a higher *aggregate* royalty share, 'fairness' having been interpreted by various participants as following a 'principle of proportionality.'[4] This can in turn lead to a less well documented dynamic phenomenon: the fact that, anticipating this *ex-post* negotiation, firms may change their strategy of producing patents, to alter their 'quality' or to contribute patents which may not be really essential for the standard in order to increase their bargaining power.[5]

Our objective in this paper is to analyze how a FRAND-like setting affects incentives to invest in R&D and the quality of the patents that firms contribute in a standard. Because the literature on patents[6] focuses mainly on their strategic use in market settings, we cannot directly rely on it to understand the effects of cooperative agreements like FRAND, especially if the focus is on the quality of the patents that will be considered essential for the standard.

The process of formation of a standard is dynamic and members (patent holders, producers of final goods and often network operators) define along the way which technologies are needed and identify which existing patents are essential for these technologies. In addition to the usual legal validity of the patent, there is also the issue of whether the patent is truly essential—from a technical point of view—for developing the standard,

[3] One of the most documented cases of dispute is the Rambus case. Rambus was involved in JEDEC (Joint Electron Device Engineering Council), the semiconductor engineering standardization body of the Electronic Industries Alliance, but later withdrew from the organization. Following a series of suits against memory manufacturers for royalty payments on the SSDRAM and DDR technologies, three firms countersued for failure to disclose these technologies during the participation in JEDEC. In the U.S. both Rambus' claim for royalty payments and the FTC request for penalties for the attempt to monopolize the market of semiconductors were eventually rejected. More germane to this paper, the European Commission launched an investigation for 'patent ambush' against Rambus leading to 'unreasonable royalties' for some technologies.

[4] More recently, proposals have been made to set a cap on royalties; see the discussion at the end of the paper.

[5] For instance, it is not rare for firms to come to the bargaining table with a small set of well identified 'strong patents'—often well known to the other members of the SSO, together with a larger set of 'other patents.' See for instance Hegde *et al*. [2009].

[6] In the following we will use 'patent' and 'patent family' interchangeably and assume that patent families have full geographical coverage.

that is whether the standard will infringe on a given patent once it is created. These specificities of SSO's introduce two important dimensions in the problem of licensing.

First, the process of verification of essentiality of the patents is difficult since the technology against which essentiality is verified is evolving. Rysmann and Simcoe [2008] show that between 1990 and 2005 the number of disclosures within some of the main SSO's has increased significantly (sometimes by a factor of five). The sheer number of disclosures makes verifiability of essentiality claims difficult.[7]

Second, the difficulty of verifying essentiality, together with the FRAND requirement that all essential patents should be disclosed, may lead firms to disclose patents that are not essential, a strategy that we call 'padding.' For instance, Goodman and Myers [2005] show that up to 80% of the patents that firms claimed to be essential for a mobile telephone standard were in fact not.[8] The tables below summarize their findings for two standards for mobile telephony ('D' denotes patents that were declared essential by the firms and 'J' those which were judged essential by the experts.

| 3GPP | D | J |
|---|---|---|
| Qualcomm | 279 | 30 |
| Ericsson | 129 | 34 |
| Nokia | 94 | 40 |
| Motorola | 38 | 11 |

| 3GPP2 | D | J |
|---|---|---|
| Qualcomm | 340 | 54 |
| Ericsson | 16 | 3 |
| Nokia | 45 | 14 |
| Motorola | 37 | 14 |

Our focus in this paper is on the consequences of the possibility of taking advantage of inessential patents. For simplicity, we assume that firms can predict which standard will arise later on and we postulate a deterministic relationship between R&D expenditures and the number of essential patents obtained.[9] But R&D generates also a series of other innovations and patents, and the firm may also decide to disclose these other patents as essential during the standard process. That is, they engage in 'padding.'

The sharing of profits depends on the *total* number of patents submitted, unless some of these are identified as inessential by the SSO, say because the

[7] In addition, there is also a lot of uncertainty as to the legal validity of patents, due in particular to randomness in the work of patent offices and as to which patents will be upheld in courts. See the book by Jaffe and Lerner [2004] on the shortcomings of the U.S. patent system and also Guellec and van Pottelsberghe [2007] on the European situation.

[8] This evaluation was made from a purely technical point of view, that is, the experts evaluated whether the standard could be produced without the technology embodied in the patent. On average, an expert spent one hour per patent to evaluate its essentiality. They use as 'essentiality' the definition provided by ETSI. 'Essential' as applied to IPR means that it is not possible on technical (but not commercial) grounds, taking into account normal technical practice and the state of the art generally available at the time of standardization, to make, sell, lease, otherwise dispose of, repair, use or operate equipment or methods which comply with a standard without infringing that IPR.

[9] Allowing for more randomness here is an interesting topic for future research, but we feel that the main results of this paper are robust to such a generalization.

technology embodied in the patent is not valuable for the standard (we will model this as disputes). To model such profit sharing, we resort to the well-known Shapley value. Beyond its simplicity, we argue that the notion of symmetry embedded in this concept is naturally compatible with the principles behind FRAND, namely (i) the fact that all downstream players are treated equally in terms of royalty rates (the ND part), and (ii) the fact that having more patents which are deemed essential for the standard allows a firm to have a higher aggregate royalty rate (the FR part). This moreover allows us to work with a pretty flexible and tractable parameterization of the shares of profit going to the upstream and downstream segments of the industry (i.e., the patent holders and the makers of the final product).

Equipped with this bargaining solution, we show that firms indeed have incentives to pad in equilibrium, and that the number of essential and inessential patents covary, since owning more inessential patents raises one's aggregate royalty share, which in turns increases the incentives to invest in R&D. Hence, if a firm pads less, it also contributes fewer essential patents. We then analyze the possibility that firms dispute the essentiality of contributed patents. We derive a 'limit padding' condition, that is, a maximum level of patents submitted such that downstream firms, when correctly anticipating the proportion of essential patents, prefer not to dispute.

In terms of welfare, we show first that when disputes are costly, and the bargaining power of the downstream firms is small, total surplus would be greater for a lower number of patents disclosed than the equilibrium level. There is therefore over-investment with respect to what a planner would like to see in the second-best situation. When disputes—or verification of essentiality—are not too expensive, there is a natural force towards the firm's reducing its number of contributed patents. When the dispute cost is small, there is under-investment by the upstream firm with respect to the second-best optimum. We identify a policy combining lower dispute costs and better filtering of patents through higher 'padding costs' that can simultaneously raise the number of essential patents while reducing (by the same amount) inessential patent contributions.

## II. MODEL

### II(i). *Firms and Markets*

There are $n + 1$ firms, denoted 0, 1, 2, . . . , $n$. Firm 0 is specialized in producing patents and is not present in the downstream market; think of this firm as being 'upstream' or as a syndicate of patent holders. Our goal is to analyze whether the upstream firm wants to pad and how changes in the costs of certification of essential patents affect the level of padding and welfare. There are $M$ downstream markets. There is only one firm present in a given market but a firm can be present in more than one market: a firm's market share is the ratio of the markets in which it is present to $M$ and is denoted by $\alpha_i$. The profit on a market is $\pi$ from the new product and is 0 otherwise.

In order for firm 0 to contribute $E$ essential patents to the future standards, it needs to invest $\varphi(E) = \mu E^2/2$ in R&D beforehand. The process of R&D generates a larger number of patents than $E$; while these extra patents are not valuable for the standard, they could be disclosed during the standards setting process as essential as long as firm 0 spends a unit cost $c$ to 'disguise' these patents as valuable for the standard.[10] To simplify, we assume that the stock of patents of the firm is large, hence that, for the equilibrium values we derive below, there are always enough inessential patents in the stock of the firm.[11]

Without loss of generality, we assume that inessential patents have no effect on profits while essential patents have: if $E$ essential patents are contributed to the standard, the realization of market profit is $\pi(E) = \pi E$. One should think of $E$ as influencing the 'quality' of the standard; note that *once* it is set, *each* of the $E$ patents is indeed essential for the standard; for a lower quality standard, involving only a subset $E'$ of the $E$ patents, only those $E'$ patents are essential for that standard.

Before turning to the analysis, we discuss the royalty rates that participants to the standard will choose for a given number of patents that are deemed essential.

## II(ii). *Fair Payoffs*

We postulate that, when there are $M$ markets and $P$ patent families, each patent earns its owner a profit of $\phi_p(P, M)$ while each individual market earns its sole supplier a profit of $\phi_m(P, M)$. For the purpose of modeling bargaining, we consider each firm as an integrator of the technologies available in its supplier network (see for instance Kranton and Minehart [2000]). There are $K$ suppliers for each firm, and the 'firm' can achieve the

[10] This is consistent with the finding in Rysmann and Simcoe [2008] that among all patents contributed to an SSO, the most cited are the most recent. If one thinks that essential patents are created explicitly in anticipation of the development of a new technological standard, as we do, and that inessential patents come from the 'stock' of existing patents (some generated during the recent R&D effort, others from previous efforts), then inessential patents come more often from the past and will also be *ex-post* cited less often in relation to the new technology.

[11] Inessential patents are therefore patents that have successfully gone through the patent office and the SSO but that are not valuable to the standard, as in the tests made in Goodman and Myers [2005]. We could extend the model to allow for patents that are 'legally' weak as in Choi [2005], Lemley and Shapiro [2005], Farrell and Shapiro [2008], and that would not be upheld in courts.

profit $\pi$ only if all the managers of its suppliers have the patents needed for their technology. The profit levels $\phi_p(P, M)$ and $\phi_m(P, M)$ are defined as:

$$\phi_p(P, M) = \frac{M}{P+K}\pi \quad \text{and} \quad \phi_m(P, M) = \frac{K}{P+K}\pi. \tag{1}$$

The appendix provides cooperative foundations for these expressions, based on the well-known Shapley value. While the Shapley value is a cooperative game theoretic concept, we view it as a convenient shortcut for modeling the outcome of a potentially complex multilateral noncooperative bargaining (e.g., Gul [1989]).

The reader should interpret the payoffs in (1) as the anticipation that the players have about the outcome of future negotiations rather than as an explicit pricing rule. Our results are robust to alternative payoff functions as long as these payoffs are increasing in the number of patents a firm contributes to the standard and in its market share in the downstream market.

Alternatively, these expressions can also be taken as a reduced form. In this perspective, beyond the convenient linear formulation, note that they imply that:

1. The 'downstream segment' is assumed to receive a total amount $MK\pi/(P + K)$ of profits while the 'upstream segment' receives a total amount $MP\pi/(P + K)$ of profits (total profits being $M\pi$). The ratio $P/K$ is therefore a measure of the relative bargaining power between the upstream and downstream industry segments. While $P$ represents the number of patents involved in the standard, $K$ is a parameter that could be 'calibrated' to replicate the upstream and downstream profit shares in a particular market; $K$ will play a crucial role in the welfare analysis. Note that as $P$ increases, the residual profit of the downstream market decreases: this is a crude illustration of 'royalty stacking.'[12]
2. In keeping with FRAND, every patent family holder receives the same royalty per unit of profit (consistent with FR), and each downstream market contributes to patent family holder revenues to the same extent (consistent with ND).
3. Following point 2, note that a rise in the number of patents (e.g., because of padding) reduces the profit of both preexisting patent owners and downstream suppliers.

As we have already noted, padding is limited by the cost of generating patents, the competition from other patent holders in the standard and the

[12] If the downstream market is oligopolistic, higher royalties would also lead to an intensification of double marginalization effects, hence a further reduction in consumer welfare (see e.g;, Lemley and Shapiro [2007]).

incentives of other parties to dispute the essentiality of patents in the standard. We turn to this now.

## III. PADDING IN THE SHADOW OF DISPUTES

A necessary condition for padding is that the set of essential patents is privately known to its owner. Since 'fair' royalties are computed on the basis of the set of patents that are deemed essential to the standard, a lower production of essential patents can be compensated by a larger production of inessential patents without detection by the other participants. This is no longer true in the case of certification or disputes if the patent holder bears the cost of disputes regarding its inessential patents.

To highlight the role of disputes, we will focus here on the leading case in which only firm 0 can produce patents and only firm 1 can dispute these patents; we think of disputes as being arbitrated within the SSO or in court. Hence, we assume implicitly that the other firms 2, . . . , $n$ are 'small' or are facing large costs of going to court.[13]

Because there is no ambiguity, we denote by $E$ the number of essential patents of firm 0, by $P$ the total number of patents contributed to the standard, hence $I = P - E$ is the number of inessential patents. Finally, we let $d$ be the proportion of patents that firm 1 decides to dispute (or the patents over which firm 1 refuses to pay the royalty).

The cost of disputing a patent is $f$ and we assume that this cost is borne by the party who loses the dispute.[14] Hence if firm 1 disputes the essentiality of a patent and the 'court' agrees, it is firm 0 which pays $f$, otherwise it is firm 1. As explained before, we assume for simplicity that only essential patents can be found essential in court. Note that the cost of one essential patent is $\mu/2$ while the industry profit per component when there is only one patent is $M\pi/K$. We assume throughout that the industry is *high profit*:

$$\frac{\mu}{2} < \frac{M\pi}{K}. \tag{2}$$

The marginal cost of disguising inessential patents into essential patents is constant and equal to $c$.

[13] Alternatively, we could consider out-of-court settlements. In this case, discoveries that some patents are inessential do not become known to the other firms: firm 0 therefore bears a lower cost in the case of dispute since it can still ask the other firms to pay the high royalty rate corresponding to the total number of patents submitted. This suggests that firm 0 will pad more. However, if in the negotiated settlement firm 1 is able to extract some of the royalty gains of firm 0 from non-disclosure, this should induce firm 1 to dispute more often. The net effect will depend on the way negotiation is modeled and on the distribution of bargaining power between firm 0 and firm 1. See Choi [1998], Farrell and Shapiro [2008] and Shapiro [2010] for models along these lines.

[14] This assumption simplifies the algebra without affecting the essence of the results.

The timing is the following:

- Firm 0 chooses $E$ and $I$ at cost $\varphi(E) + cI$.
- Firm 1 observes the total number of patents $P = E + I$ and decides on the proportion $d$ of patents to dispute.
- If after a dispute there are $P'$ patents that have not been found inessential, royalty payments are decided on the basis of condition (1) with $P = P'$ patents.

An equilibrium is a pair $(E, I)$ for firm 0 and a binary decision $d \in \{0, 1\}$ by firm 1 to dispute or not dispute the patents of firm 0.

We proceed as follows:

- We first characterize the optimal choice of essential patents for a given total number of patents $P$ assuming that firm 1 will not dispute the patents. We show that there is padding whenever the total number of patents is greater than a cutoff level $P^{eq}$;
- As $P$ increases, the numbers of essential patents and of inessential patents increase and there exists a cutoff value $P^{lim} > P^{eq}$ at which firm 1 is indifferent between disputing or not disputing, given the optimal choice of firm 0.
- A revealed preference argument shows that there cannot be an equilibrium where firm 1 disputes patents with probability one.
- We then derive the optimum choice of firm 0, that is, the number of patents it will effectively produce for the standard and show that there is 'limit padding' in the sense that firm 0 produces the number of patents that make firm 1 indifferent between disputing or not disputing the patents.[15]

*Choice of Essential Patents When Firm 1 Does Not Dispute*

Assuming that firm 1 does not dispute any patent when there are $P$ patents put forward by firm 0, the optimal choice of essential patents by firm 0 solves:

$$\max_{E \leq P} \frac{P}{P+K} M\pi E - \mu \frac{E^2}{2} - c(P-E).$$

Ignoring the constraint $E \leq P$, the unconstrained maximum is achieved at:

[15] Moreover, firm 1 does not randomize and uses the pure strategy of not disputing. This result is due to our assumption that disputes happen before the profit is known. If disputes can arise after firm 1 gets information about the market profit, there will be a dispute for high levels of profit and no dispute for low levels of profit: this is apparent by inspecting (5) below and interpreting $\pi$ as the realized profit. We chose our timing because it is empirically reasonable (market profits are realized well after the royalty agreements are made) and because it captures in a simple way the role of court fees on padding behavior.

$$\sigma(P) = \frac{1}{\mu}\left\{\frac{P}{P+K}M\pi + c\right\}. \tag{3}$$

The constrained maximum is $E(P) = \min\{\sigma(P), P\}$. A first observation is that while padding $P - \sigma(P)$ is decreasing in $c$, padding is bounded above even if $c = 0$, that is when there is no cost of disguising patents as essential. A second observation is that there is no padding—that is $E(P) = P$—if and only if $P$ is smaller than a cutoff value (note that all proofs missing in the text appear in the appendix):

*Proposition 1.*

(i) There exists $P^{eq} > 0$ such that the optimal number of essential patents at $P$ assuming that firm 1 does not dispute is:

$$E(P) = \begin{cases} P & \text{if } P \leq P^{eq} \\ \sigma(P) & \text{if } P \geq P^{eq}. \end{cases}$$

(ii) $E(P)$ is increasing and concave in $P$,
(iii) $P - E(P)$ is increasing in $P$, $(P - E(P))/P$ is increasing and concave in $P$.

In other words, without disputes, there is padding only for $P > P^{eq}$ and the level of padding is increasing in $P$, both in absolute number but also relative to the number of patents. Everything else being equal, a larger number of patents indicates a higher proportion of inessential patents.

*Limit Padding*

Consider now the behavior of firm 1. If, at $P$, firm 1 has beliefs $E$, and disputes a proportion $d$ of patents, while firm 0 actually chooses a number $\hat{E}$ of essential patents, the payoffs for firms 0 and 1 are as follows:

$$u_0\left(\hat{E}, d; P, E\right) = \frac{\hat{E} + (1-d)(P-E)}{\hat{E} + (1-d)\left(P - \hat{E}\right) + K} M\hat{E}\pi - fd\left(P - \hat{E}\right) - \frac{\mu \hat{E}^2}{2} - c\left(P - \hat{E}\right)$$

$$u_1(E, d; P) = \frac{\alpha_1 K}{E + (1-d)(P-E) + K} ME\pi - fdE. \tag{4}$$

Note that if firm 1 disputes the patents, it creates an industry wide externality since it becomes known which patents are truly essential. This externality translates into lower royalty payments for all firms.[16]

[16] Hence, disputing the patents of firm 0 has the flavor of a public good, as noted also by Farrell and Shapiro [2008]. If more than one firm can dispute patents, there is a free rider problem since each firm will prefer another firm to dispute in order to benefit from the externality without having to bear the court costs. We should expect that, in equilibrium, the

Since $u_1(E, d; P)$ is convex in $d$, the best response of firm 1 is either to dispute all patents ($d = 1$) or to dispute no patent ($d = 0$). It is best for firm 1 not to dispute if and only if:

$$\text{(5)} \qquad \frac{P-E}{(E+K)(P+K)}\alpha_1 M\pi - f \leq 0.$$

In order to avoid disputes, firm 0 must choose a minimum number of essential patents. However because firm 1 observes only $P$ and not $E$, the condition must be met at the optimal choice for firm 0.

Can we have an equilibrium in which firm 1 disputes all $P$ patents with probability one? In such a case, firm 0 will still choose $\hat{E}$ in order to maximize the payoff given by (4). However, if $E^d(P)$ is the optimum, it must be the case that $P = E^d(P)$, that is, firm 0 should abstain from padding: It would rationally expect that such padding would be successfully undone by the dispute and would only imply a cost of $f(P - E^d(P))$ for firm 0. But if, in equilibrium, there are no inessential patents, it will be anticipated by firm 1, which will find it optimal not to dispute the patents, which is a contradiction. We therefore have the following result:

*Lemma 1.* In equilibrium, the padding constraint (5) is satisfied, so that disputes do not arise.

When $P > P^{eq}$, by substituting $\sigma(P)$ for $E$ in (5) the no-dispute condition becomes, $\Delta(P) \leq f$ where:

$$\Delta(P) \equiv \frac{P-\sigma(P)}{(\sigma(P)+K)(P+K)}\alpha_1 M\pi$$

is the difference in per-patent payoffs to firm 1 when it does not dispute patents, and when it does dispute patents. For a given total number of patents, the number of essential patents is the same in the two cases but when there is dispute, firm 1 will only pay royalties on the number of essential patents. We show in the Lemma A2 in the Appendix that $\Delta(P)$ is increasing in $P$ in the padding region $P \geq P^{eq}$ and is bounded above by $\Delta(\infty) = \mu \frac{\alpha_1 K M\pi}{M\pi + c + \mu K}$.

A necessary condition for firm 1 to be willing to dispute is that $\Delta(P)$ is less than the unit cost of dispute $f$. There are therefore two cases of interest: the case of a high dispute fee, when the unit cost of disputes is larger than the upper bound $\Delta(\infty)$, and the case of a low dispute fee when $f$ is smaller than $\Delta(\infty)$. It is only in the second case that disputes can effectively put pressure

level of patents for which a given firm is indifferent between disputing or not disputing will increase: in other words, the process of *ex-post* certification by disputes will be even less efficient in preventing padding than under our assumption.

on padding. In the case of a low dispute fee, since $\Delta(P)$ is increasing in $P$, there exists a unique value $P^{lim}$ such that:

$$\Delta(P^{lim}) = f.$$

From (iii) of Proposition 1, the value of $P^{lim}$ for which (5) binds is an increasing function of $f$. This is indeed intuitive: decreasing the cost of disputes will make firm 1 more aggressive in disputing patents when $P$ is greater than $P^{eq}$.

## IV. EQUILIBRIUM PADDING AND WELFARE

The policy implications of the model are nontrivial because of the covariation between essential and inessential patents. This covariation further suggests that it is possible for the upstream firm to *over-invest* in R&D in order to increase its share of the industry profits. Indeed, while in standard moral hazard situations, there is a tendency for under-investment because the agent does not get the full marginal return from its investment, in our model there is an additional effect at play, due to the dependence of the share on the contribution of firm 0: the more the firm contributes, the higher its share of the return. As we shall see, the second effect—in the absence of disputes—dominates the first effect and leads to over-investment from a social point of view.

### IV(i). *High Dispute Fee*

In this case, there is no dispute for any value of $P$. We first show that as long as $c$ is small enough, there is padding in equilibrium, that is, the firm chooses to invest $E > P^{eq}$:

*Proposition 2.*

(i) There exists $\hat{c} > 0$ such that whenever $c < \hat{c}$, firm 0 chooses optimally to produce inessential patents.
(ii) The number of patents $P^{no}$, $P^{no} > P^{eq}$ solves:

$$\sigma(P^{no}) = \frac{c(P^{no} + K)^2}{KM\pi}.$$

In the region with padding, the payoff to firm 0 is given by:

$$u_{0+}(P) = \frac{P}{P+K} M\pi\sigma(P) - \mu\frac{\sigma(P)^2}{2} - c(P - \sigma(P)).$$

By contrast, a social planner, maximizing total welfare, would in this second-best world choose a number of patents $P$ solving:

$$\max_P W(P) \equiv u_{0+}(P) + \frac{K}{P+K} M\pi\sigma(P).$$

The marginal welfare is then:

$$W'(P) = u'_{0+}(P) + \frac{KM\pi}{P+K}\left(\sigma'(P) - \frac{\sigma(P)}{P+K}\right).$$

Now, direct computations show that:

$$\sigma'(P) - \frac{\sigma(P)}{P+K} = \frac{M\pi}{\mu(P+K)}\left(\frac{K-P}{P+K} - c\right)$$

which is negative if, and only if:

(6) $(1+c)P > (1-c)K.$

A sufficient condition for this is $P > K$.

A necessary condition for a social optimum is that $W'(P) = 0$, and therefore under (6) we need to have $u'_{0+}(P) > 0$, implying—we show in Lemma A3 in the Appendix that $u_{0+}(P)$ is single-peaked—that a (second-best) social optimum requires a number of patents less than $P^{no}$: from a social point of view *less padding* would be desirable. As $K$ decreases, there are two effects. First, the number of essential patents for a given number of patents increases (since $\sigma(P)$ is decreasing in $K$); this is the usual effect that giving a larger share $(P/(P + K))$ of the profits to the innovator leads to higher innovation. But because of the co-variation between $E$ and $P$ the optimal number of patents in Proposition 2 also increases. Hence, as $K$ decreases, it is clear that the condition $P^{no} > K$ will be satisfied. Intuitively, it is when the hold-up problem is the most severe ($K$ small) that padding is excessive with respect to the second-best surplus optimum. We can summarize these results in the following corollary:

*Corollary 1.*

(i) There exists $\bar{K}$ such that for all $K < \bar{K}$, $P^{no} > K$.

(ii) If a planner could impose a limit on the number of patents submitted to the standard, then, whenever $K < \bar{K}$, he would choose a lower number of patents than firm 0.

An indirect instrument for reducing the level of padding would be to raise the cost $c$ for firm 0 to push a patent through the SSO, for instance to be 'certified'.[17] Nevertheless the consequences of this are not immediate because of two opposite forces. First, since inessential patents become more costly to disclose, there will be a substitution towards essential patents, suggesting an improvement in the R&D effort. However, *given the new cost* $c$, the covariation between essential and inessential patents suggests a decrease in the R&D effort. The next proposition stresses that the second effect dominates:

*Proposition 3.* Suppose that $c < \hat{c}$ and that the equilibrium number of patents is $P^{no}(c)$. Then, locally, as $c$ increases, the numbers of both essential and inessential patents decrease.

In terms of social welfare, as long as the bargaining power of the downstream firms is low ($P^{no} > K$), welfare will also increase when $c$ increases. Indeed, as $c$ increases, firm 0 will contribute fewer patents by Proposition 3 while by Corollary 1 we know that *for the same level of* $c$ welfare is locally increasing when $P$ decreases. Hence the overall effect of an increase in $c$ is positive for welfare.[18]

### IV(ii). *Low Dispute Fee*

Because firm 1 has no reason to dispute when $P \leq P^{eq}$, firm 0 will always pad. However disputes may discourage padding since the no-dispute condition (5) must be satisfied. Because the payoff to firm 0 when there is no dispute is single peaked, firm 0 optimally chooses to disclose a number of patents $P^d = \min\{P^{lim}, P^{no}\}$. Disputes effectively reduce padding only if $P^{lim} < P^{no}$, and in this case firm 1 does not dispute, even if it is indifferent between disputing and not doing so. This is because disputes create a first order cost for firm 0 which would therefore infinitesimally reduce its total number of patents to make it strictly optimal for firm 1 not to dispute.

*Proposition 4.* Suppose that dispute fees are low and that the cost of inessential patents is low ($c < \hat{c}$). Then, firm 0 produces $\min(P^{lim}, P^{no})$ patents and there is no dispute in equilibrium.

If firm 0 produces $P^{no}$ patents, we have the same welfare result as in the previous case, and there can be over-investment relative to the second-best, i.e., when the planner could impose the number of patents but padding cannot be avoided. This over-investment is due to the fact that the share of

[17] Obviously, if the planner had full information about the environment, he could simply set a bound on the number of patents. However, a more realistic assumption is that the planner does not have perfect information about some parameters and therefore a policy based on the number of patents is likely to lead to even more inefficiencies. Since the effect of an increase in $c$ is unambiguous from Proposition 3, it is a more reasonable policy instrument.

[18] Alternatively, by the chain rule, $dW(P)/dc = (\partial P/\partial c)(\partial W/\partial P)$; the first term is negative by Proposition 3 and the second is also negative when $K$ is small by Corollary 1.

profits is increasing in the number of contributed patents and that padding is an economical way to do so for firm 0, but also reinforces the incentives to invest in *R&D*.

The over-investment effect is obviously reduced when disputes are effective. As we now show, when disputes become very effective ($f$ is close to zero), and when padding is not costly, firm 0 will tend to under-invest.

Consider the situation where firm 0 is effectively constrained by the threat of disputes and produces $P^{lim}$. As $f$ becomes close to 0, it is clear that $P^{lim}$ converges to $P^{eq}$ and therefore to $\sigma(P)$: there is no padding in equilibrium. Using $c$ close to zero in (3), $P^{lim} = \frac{1}{\mu} M\pi - K$. If there is no padding, the planner would maximize $M\pi E - \mu E^2/2$ and therefore would choose to set $E = M\pi/\mu$. Hence, when $c$ and $f$ are close to zero, firm 0 indeed under-invests in R&D. As $f$ becomes larger, we could have the over-investment effect of the previous case.

As $c$ increases, the downstream firm disputes less aggressively, hence the limit padding constraint of the upstream firm is weakened and firm 0 will *increase* the number of patents it brings to the standard: contrary to the case where disputes are ineffective, with limit padding, higher costs of certification *increase* the number of patents that firm 0 contributes when there is limit padding.

As $f$ decreases, the downstream firm finds it less costly to dispute patents and the limit padding constraint is strengthened, leading to a decrease in the number of inessential patents that are submitted. However, by the covariation result, it is also the case that the number of essential patents (or R&D investment) decreases.

*Proposition 5.* Suppose that $c < \hat{c}$ and that the equilibrium number of patents is $P^{lim}(c, f)$.

(i) As $f$ decreases locally, the total number of patents, the number of essential patents and the number of inessential patents decrease.
(ii) As $c$ increases locally, the total number of patents, the number of essential patents and the number of inessential patents increase.

Whether or not an increase in $c$ or a decrease in $f$ is welfare improving depends on whether there is initially over or under-investment. For instance as $f$ is close to zero, there is under-investment and therefore an increase in $c$ will yield *more* essential patents and is welfare improving but a decrease in $f$ will lead to *fewer* essential patents and is therefore welfare depressing. Opposite effects arise when there is over-investment.

However, if the goal is to improve the mix of essential/inessential patents that firm 0 contributes to the SSO, one can show that a policy that *combines* a decrease in $f$ with an increase in $c$ will achieve this goal.

*Proposition 6.* Suppose that $c < \hat{c}$ and that the equilibrium number of patents is $P^{lim}(c, f) < P^{no}(c)$. Then, there exist $dc > 0$ and $df < 0$ such that the number of patents stays constant but the proportion of essential patents increases.

## V. DISCUSSION

This paper has considered the case of a single firm involved in patenting. In our working paper (Dewatripont and Legros [2008]), we also consider an extension wherein an upstream firm and a vertically-integrated firm are both involved in patenting. There, a key result when dispute costs are high is that the two firms, if equipped with the same patent production technology, will produce the same number of essential patents but the upstream firm will pad more than the vertically-integrated firm. The overall incentive to submit patents is indeed higher for the upstream firm because more patents mean more money for the upstream industry segment but less money for the downstream industry segment. Firms that are also active in the downstream market have therefore a lower incentive to submit patents. As dispute costs decrease, the limit padding condition for the upstream firm starts binding while it is not binding for the vertically integrated firm. This implies that the upstream firm will now produce fewer essential patents than the vertically integrated firm.

These results seem consistent with empirical results obtained by Goodman and Myers [2005] for the case of mobile telephone standards. Indeed, they show that, while all major patent producers seem to exaggerate claims of essentiality, the extent of exaggeration seems to be much more significant in the case of a firm like Qualcomm, which is 'more upstream' than its main rivals.[19] While this deserves further investigation, this is evidence consistent with our analysis.

Coming back to the results of this paper, by abandoning the usual assumption that patents bring known benefits to the industry or that their benefits are known to all parties, we have shown that the threat of disputes reduces incentives to pad but at the cost of lower production of essential patents. This result has potentially significant policy implications, calling for combining easier certification with an increase in the cost of padding, that is, better filtering of patent submissions. Looking for an 'operational' way of limiting padding while simultaneously encouraging innovation constitutes an interesting avenue for further research. While this model is a useful starting point in this respect, it would benefit from extensions.

Some of our assumptions could indeed be relaxed. As long as there are no disputes, the model behaves in the same way if there is a stochastic R&D technology. Introducing disputes in a stochastic environment raises additional difficulties however. It would also be reasonable to link the level of

[19] Geradin *et al.* [2008] classify in their analysis Qualcomm as an upstream firm and Nokia, Ericsson and Motorola as vertically integrated.

R&D to the number of inessential patents that a firm can contribute. If more R&D implies both more essential but fewer inessential patents, the relationship between R&D level and padding may be non-monotonic. Note that the quality of the patent office will matter here since it will affect the stock of existing patents that could be used as inessential patents.[20]

The patent dispute process could be generalized, to allow for the optimality of 'partial disputes', i.e., on a subset of the patent portfolio.

And, very importantly, we have kept the choice of standard in 'reduced form', while it would be very interesting to link this choice explicitly to the outcome of the R&D investment process. These extensions are beyond the scope of this paper but constitute interesting avenues for future research.

Finally, while FRAND is one answer to the issue of hold-up, we have shown that its by-product, padding, leads to a reduction in the share of profits flowing to the downstream firms, or more generally to the users of the standard. While we have taken as given here that the standard will be used, a low share of downstream profits may discourage users from adopting the standard. In ongoing research we show that caps on aggregate royalties may help the SSO commit to give a reasonable share of profits to end users, but caps also create incentives for firms not to join the SSO. There is therefore a non trivial tradeoff between bringing firms and end users on board within the SSO.

## APPENDIX

### *A.1. Fair Payoffs: Foundation*

We show in this appendix that the payoffs expressed in (1) can be obtained using the Shapley value (see Myerson [1977] for a general theoretical discussion; Hart and Moore [1990] for an application to incomplete contracting; Layne-Farrar *et al.* [2007] for an application to mobile-phone patents.) Specifically, assume that:

- There is a set $\mathcal{P}$ of patents that are claimed to be essential to the production of a product. Since each patent is assumed to be controlled by one manager, they all have to have agreed to license their patent in order for production to go ahead.
- There are $K$ managers in the supplier network in a given market, hence there are $MK$ managers; all managers in one market are strict complements in the sense that their firm can produce the new product only if all managers have contracted with the patent holders.

By assuming extreme decentralization of both the patent decisions and the production decisions, we in fact parameterize the relative bargaining powers of the upstream and downstream segments of the industry.

[20] The quality of the patent office is endogenous since it reflects the incentive scheme put in place for patent examiners, see e.g., Friebel *et al.* [2006] for the role of incentives at the European PTO.

The Shapley value is defined axiomatically, and its symmetry-based fairness is in keeping with FRAND in the following sense. First, the 'nondiscriminatory' ingredient in FRAND can be interpreted as requesting that the royalty paid by firms is the same for each individual market, independently of market share; second, the 'fair and reasonable' ingredient can be interpreted as requesting that patent holders receive the same per-patent royalty independently of their total portfolio.

This also means that each patent and each market is treated as a separate entity. Hence there are effectively $P + MK$ players: $P$ patent holders and $K$ managers per market. Because of symmetry, we know that the payoff to a manager in each market is the same, and that the payoffs to the patents are the same.

Remember that if for a given set of players $\mathcal{N}$, the total payoff to a coalition $E$ is described by a function $v(E)$, the Shapley value is defined as follows. Consider a random order on $\mathcal{N}$, let $S_i$ be the set of all players preceding $i$ in this random order. The marginal contribution of $i$ is $v(S_i \cup \{i\}) - v(S_i)$, and the Shapley value is given by:

$$\phi_i = E\left[v(S_i \bigcup \{i\}) - v(S_i)\right]$$

where $E$ is the expectation operator when all $|N|!$ orders over $\mathcal{N}$ are assigned equal probability. It follows that if two players have the same marginal contributions if they occupy the same position in an order that their Shapley values be the same.

Consider the case of a unique market, hence when there are $P + K$ players. Since firms need all the patents in $\mathcal{P}$ in order to use the standard, and since the profit of a firm is realized only if its $K$ managers have acquired the patents, any coalition not containing all the patents or all the managers would not produce a profit. Hence each patent and each manager has a positive marginal contribution of $\pi$ if and only if it is 'last' in the order on the $P + K$ players. It follows that each manager and each patent holder receives $\pi/(P + K)$. Since on a market the $K$ managers belong to the same firm, the firm receives $K\pi/(P + K)$.

We prove now that this value is the same independently of the number $M$ of markets:

*Lemma A1.* Consider $P$ essential patents and $M$ markets, the Shapley value for each patent and for a firm present on a market are respectively:

$$\phi_p(P,M) = \frac{M\pi}{P+K} \quad \textit{and} \quad \phi_m(P,M) = \frac{K\pi}{P+K}. \tag{7}$$

*Proof.* We index patents by $i = 1, \ldots, P$ and we index managers by $k(m, i)$, $i = 1, \ldots, K$, $m = 1, \ldots, K$ where $k(m, i)$ is the i-th manager present on market $m$. By assumption, all managers $k(m, i)$, $i = 1, \ldots, K$ belong to the same firm. Let $\mathcal{K}_{\mathcal{M}}$ be the set of managers.

By symmetry, we know that all patents have the same Shapley value $\phi_p$ and all managers in market $m$ have the same Shapley value. Hence, if there are $N$ players, it is enough to determine the Shapley value $\phi_p(\mathcal{N})$ common to all patent holders and the Shapley value $\phi_{j(m,k)}(\mathcal{N})$ of all the $K$ managers in market $m$, where $m = 1, \ldots, M$. The value of a firm present in a market is then $K\phi_{j(m,k}(\mathcal{N})$

We use the following property of the Shapley value : balanced contribution (Myerson [1977]) requires that what player $i$ contributes to player $j$ be equal to what player $j$ contributes to player $i$. Or if $\mathcal{N}$ is the set of players, that:

$$\phi_i(\mathcal{N}) - \phi_i(\mathcal{N} - \{j\}) = \phi_j(\mathcal{N}) - \phi_j(\mathcal{N} - \{i\}).$$

In our case, $\mathcal{N} = \mathcal{P} \bigcup \mathcal{K}_{\mathcal{M}}$. Letting $i$ be a patent and $j(m, k)$ be a manager, we know that $v(\mathcal{N} - \{i\}) = 0$ since no new product can be put on the market if one essential patent is missing and $v(\mathcal{N} - \{j(m,k)\}) = (M-1)\pi$ since if one component is not present, a product cannot be produced on market $m$.

In the game $\mathcal{N} - \{j(m,k)\}$ all players $j(m, k')$ have zero marginal contributions, hence their Shapley value is equal to zero: $\phi_m(\mathcal{N} - \{j(m,k)\}) = 0$. It follows that balanced contribution implies:

$$\phi_p(\mathcal{N}) - \phi_p(\mathcal{N} - \{j(m,k)\}) = \phi_{j(m,k)}(\mathcal{N}), \text{ for each } m, k. \tag{8}$$

To show (7), we proceed by induction on the number of markets $M$. From the text, the result is true for $M = 1$. We suppose it is true for $M - 1$ and we show that the result is true for $M$.

Since for each coalition $E$, $v(S - \{j(m,k)\}) = v(S - \bigcup_{k=1}^{K}\{j(m,k)\})$, the marginal contributions of all players are the same when the set of players is $\mathcal{N} - \{j(m,k))\}$ and when it is $\mathcal{N} - \bigcup_{k=1}^{K}\{j(m,k)\})$. In the later case, the game is in fact the one with $M - 1$ markets and the Shapley values are given by (7).

In the initial game with $\mathcal{N}$, all managers are symmetric and therefore have the same value. Hence, by efficiency, we have :

$$P\phi_p(\mathcal{N}) = M\pi - MK\phi_{j(m,k)}(\mathcal{N}). \tag{9}$$

Hence, (7), (8) and (9) imply :

$$\phi_{j(m,k)}(\mathcal{N}) = \frac{M\pi - MK\phi_{j(m,k)}(\mathcal{N})}{P} - \frac{(M-1)\pi}{P+K}$$

$$\Leftrightarrow \quad \phi_{j(m,k)}(\mathcal{N}) = \frac{\pi}{P+K}$$

implying as claimed that $\phi_m(\mathcal{N}) = \frac{K\pi}{P+K}$ proving the induction hypothesis and the lemma. □

### A.2. *Proof of Proposition 1*

Differentiation yields:

$$\sigma'(P) = \frac{M\pi}{\mu(P+K)} \frac{K}{P+K},$$

which is positive while $\sigma''(P)$ is negative; hence $\sigma(P)$ is an increasing and concave function of $P$. Since $\sigma(0) = c/\mu$ while $\sigma(\infty) = (M\pi + c)/\mu$, there exists a unique $P$ solving $\sigma(P) = P$ proving (i).

Because $E(P) = \min\{\sigma(P), P\}$, $E(P)$ is increasing and concave in $P$, proving (ii).

Now, when $P > P^{eq}$:

$$\begin{aligned}\sigma'(P) &= \frac{1}{\mu}\frac{M\pi}{P+K}\frac{K}{P+K} \\ &< \frac{1}{\mu}\frac{M\pi}{P+K} \\ &< \frac{P-c/\mu}{P} \\ &< 1,\end{aligned}$$

where the first inequality is due to $\frac{K}{P+K} < 1$; the second to the fact that $P > P^{eq}$, $\sigma(P) < P$ and therefore $\frac{1}{\mu}\frac{M\pi}{P+K} < \frac{P-c/\mu}{P}$. Therefore $P - \sigma(P)$ is increasing in $P \geq P^{eq}$ and therefore $P - E(P)$ is increasing in $P$.

Finally, differentiation shows that $\sigma(P)/P$ is decreasing and convex in $P$, which implies (iii).

### *A.3. Properties of Δ(*P*)*

*Lemma A2.*

1. If $\Delta(\infty) \leq f$, then (5) is satisfied for all values of $P$.
2. If $\Delta(\infty) > f$, then there exists a unique value $P^{lim} > P^{eq}$ such that (5) binds.
3. $\Delta'(P) > 0$ for all $P > P^{eq}$.

Differentiating $\Delta(P)$, we have:

$$\begin{aligned}\Delta'(P) &= \alpha_1 K M\pi\left\{-\frac{\sigma'(P)}{(\sigma(P)+K)^2} + \frac{1}{(P+K)^2}\right\} \\ &\propto -\frac{K}{(P+K)^2}\frac{M\pi}{\mu}\frac{1}{(\sigma(P)+K)^2} + \frac{1}{(P+K)^2} \\ &\propto \frac{1}{(P+K)^2}\left[1 - \frac{M\pi}{\mu}\frac{K}{(\sigma(P)+K)^2}\right].\end{aligned}$$

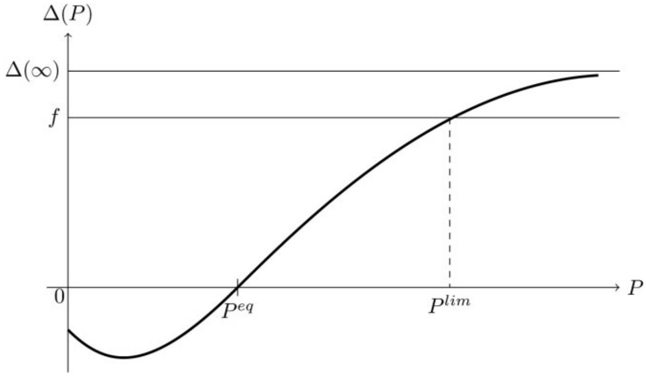
The term in brackets is an increasing function of $P$ since $\sigma(P)$ is an increasing function of $P$. Hence, the sign of $\Delta'(P)$ is 'increasing' in $P$: if $\Delta'(P) > 0$, then $\Delta'(P') > 0$ for all $P' > P$.

We note that $\Delta(0) < 0$ (since $\sigma(0) > 0$), $\Delta(P^{eq}) = 0$, and that $\Delta(\infty) = \mu\frac{\alpha_1 K M\pi}{M\pi + c + \mu K}$. Hence, there is a cutoff value of $P$ such that $\Delta'$ is positive only when $P$ is larger than this cutoff value.

Now, if $\Delta(\infty) < f$, $\Delta(P) < f$ for all $P$ and (i) follows.

If $\Delta(\infty) > f$, there exists a unique value—strictly greater than $P^{eq}$—such that $\Delta(P) = f$, and at this value $\Delta(P)$ must be increasing in $P$, proving (ii) and (iii).

The function $\Delta(P)$ has typically the following graph.[21] We have also illustrated the case (ii) of the lemma where the limit padding constraint binds.[22]




*A.4. Proof of Proposition 2*

It is convenient to consider the cases where $P$ is lower and greater than $P^{eq}$.

When $P \leq P^{eq}$, there is no padding and firm 0 chooses $P$ to maximize $u_{0-}(P) = \frac{P^2}{P+K} M\pi - \mu \frac{P^2}{2}$. The derivative of this function is:

$$(10) \qquad u'_{0-}(P) = \frac{P^2 + 2PK}{(P+K)^2} M\pi - \mu P$$

and the sign of the derivative is the same as the sign of $\frac{P+2K}{(P+K)^2} M\pi - \mu$. This function is decreasing in $P$ and has value $\frac{2M\pi}{K} - \mu$ at $P = 0$, and is therefore positive by (2). As $P \to \infty$, the function has limit equal to $-\mu$. It follows that:

*Lemma A3.* The function $u_{0-}(P) = \frac{P^2}{P+K} M\pi - \mu P$ is single peaked.

*Proof.* Since the sign of the derivative is first positive and then negative, $u_{0-}(P)$ is single peaked. The zero of the derivative is attained at the positive root of $\mu P^2 + (4\mu K - M\pi)P + 4\mu K^2 - 2KM\pi = 0$. Simple algebra shows that the positive root is $\frac{1}{2\mu}\left(M\pi - 2K\mu + \sqrt{M\pi(M\pi + 4K\mu)}\right)$. □

This optimal value of $P$ can be smaller or greater than $P^{eq}$ depending on the value of $c$. If $c$ is 'small' however, the optimum is greater than $P^{eq}$ and therefore it is optimal for firm 0 to choose $P = P^{eq}$.

*Corollary A1.* There exists $\hat{c} > 0$ such that for all $c < \hat{c}$, the maximum payoff to firm 0 over $P \leq P^{eq}$ is attained at $P^{eq}$.

[21] It may not be decreasing at zero, but is always increasing beyond $P^{eq}$.
[22] Obviously the economically relevant part of the graph is when $P \geq P^{eq}$.

*Proof.* From the definition of $P^{eq}$, we have:

$$(11)\qquad \mu = \frac{M\pi}{P^{eq}+K} + \frac{c}{P^{eq}}.$$

If $S(P) = P$, $u_0'(P)=0$ at $\hat{P}$ such that:

$$(12)\qquad \mu = \frac{M\pi}{\hat{P}+K} + \frac{2KM\pi}{(\hat{P}+K)^2}.$$

Note that in (12), $\hat{P} > \frac{M\pi}{\mu} - K$ and that $\hat{P}$ is not a function of $c$. When $c = 0$, $P^{eq} = \frac{M\pi}{\mu} - K$, and therefore $P^{eq} < \hat{P}$ for low enough values of $c$. Precisely, the result holds for all $c \leq \hat{c}$ such that $\mu \geq \frac{M\pi}{\hat{P}+K} + \frac{c}{\hat{P}}$ for then it is necessary to set $P^{eq} \leq \hat{P}$ in order to restore the equality in (11). The condition is equivalent to having $c$ lower than a cutoff level $\hat{c}$:

$$\hat{c} = \hat{P}\left(\mu - \frac{M\pi}{\hat{P}+K}\right) \qquad \square$$

Consider now the case $P > P^{eq}$. There is padding and firm 0 chooses $P$ to maximize:

$$u_{0+}(P) = \frac{P}{P+K} M\pi\sigma(P) - \mu\frac{\sigma(P)^2}{2} - c(P - \sigma(P)).$$

By the envelope theorem,

$$u_{0+}'(P) = \frac{K}{(P+K)^2} M\pi\sigma(P) - c.$$

From proposition 1, the ratio $\sigma(P)/(P + K)^2$ is decreasing in $P$. Hence the sign of $u_{0+}'(P)$ is 'decreasing' and $u_{0+}(P)$ is single peaked. While there is a change of regime at $P^{eq}$, it is possible to show that the payoff is differentiable at $P^{eq}$,[23] and therefore that for $c < \hat{c}$, $u_{0+}'(P^{eq}) = u_{0-}'(P^{eq})$ is positive. It follows that there is an interior solution $u_{0+}'(P) = 0$. Because $P = P^{eq}$ is feasible, this interior solution is also a global maximum.[24]

## *A.5. Proof of Proposition 3*

Since we will be doing comparative statics with respect to $c$, $f$ in the following Propositions, we make explicit the dependence of $\sigma$, $P^{lim}$ on these parameters.

[23] To see this, use $P^{eq} = \frac{1}{\mu}\{\frac{P^{eq}}{P^{eq}+K} M\pi + c\}$ and substitute $\mu = \frac{M\pi}{P^{eq}+K} + \frac{c}{P^{eq}}$ in $u_{0-}'$.

[24] When the cost of producing inessential patents, $c$, is instead greater than the cutoff $\hat{c}$, firm 0 will not pad: the optimum when $P \leq P^{eq}$ is strictly lower than $P^{eq}$, implying $u_0'(P^{eq}) < 0$, while the optimum when $P \geq P^{eq}$ is at $P^{eq}$, implying a global maximum less than $P^{eq}$. This is rather intuitive: inessential patents increase the revenue only through the level of royalties but not through the level of market profits: if the cost of inessential patents is high enough, they become a poor substitute for essential patents in raising total profits since essential patents increase both the royalty rate and the market profit $\pi$.

The implicit function theorem applied to to the expression in Proposition 3 implies that the sign of $dP^{no}(c)/dc$ is the same as the sign of:

$$\begin{aligned}
&\frac{K}{(P^{no}+K)^2}M\pi\frac{\partial\sigma(P^{no},c)}{\partial c}-1\\
&=\frac{K}{(P^{no}+K)^2}M\pi\frac{1}{\mu}-1\\
&=\frac{c}{\mu\sigma(P^{no},c)}-1\\
&<0,
\end{aligned}$$

where the last equality follows the definition of $P^{no}(c)$ and the inequality the definition of $\sigma(P, c)$.

### *A.6. Proof of Proposition 5*

We denote the partial derivative of $\sigma(P, c)$ with respect to $P$ by $\sigma_P(P, c)$. The proof proceeds in two steps:

(i) Remember that $P^{lim}(c,f)$ solves $\Delta(P, c) = f$. As $f$ increases, the left hand side must increase; because the left hand side is increasing in $P$ (Lemma A2), it follows that $P^{lim}(c, f)$ is increasing in $f$. Since the number of essential patents $\sigma(P, c)$ is independent of $f$ and is increasing in $P$, the number of essential patents also increases. Finally, the variation of padding is:

$$\frac{\partial}{\partial f}\left(P^{lim}-\sigma(P^{lim},c)\right)=\frac{\partial P^{lim}(c,f)}{\partial f}\left(1-\sigma_P(P^{lim},c)\right),$$

which is positive if $\sigma_P(P^{lim}, c)$ is less than one. However, since $P^{lim} > P^{eq}(c)$ and since $P^{eq}(c)$ intersects the diagonal from above, the slope at $P^{eq}$ is less than unity. By concavity of $\sigma(P, c)$ in $P$ it is also the case that $\sigma_P(P^{eq}(c), c)$ is less than unity.

(ii) As $c$ increases, $\sigma(P, c)$ increases by $1/\mu$; hence $\Delta(P, c)$ decreases. To restore the equality $\Delta(P, c) = f$ it is necessary to increase $P$, proving that $P^{lim}(c, f)$ increases with $c$. To facilitate the exposition we will write $P^{lim}$ instead of $P^{lim}(c, f)$. For essential patents,

$$\frac{d\sigma(P^{lim},c)}{dc}=\frac{\partial P^{lim}}{\partial c}\sigma_P(P^{lim},c)+\frac{1}{\mu},$$

which is positive since $P^{lim}(c, f)$ is increasing in $c$. For inessential patents:

$$\frac{d(P^{lim}-\sigma(P^{lim},c))}{dc}=\frac{\partial P^{lim}}{\partial c}\left(1-\sigma_P(P^{lim},c)\right)-\frac{1}{\mu}.$$

By the implicit function theorem:

$$\begin{aligned}\frac{\partial P^{lim}}{\partial c} &= -\frac{\partial \Delta / \partial c}{\partial \Delta / \partial P} \\ &= -\frac{\frac{-1}{\mu(\sigma+K)^2}}{\frac{-\sigma_P}{(\sigma+K)^2}+\frac{1}{(P+K)^2}} \\ &= \frac{1}{\mu}\frac{(P+K)^2}{(\sigma+K)^2-\sigma_P(P+K)^2} \\ &> \frac{1}{\mu}\frac{1}{1-\sigma_P}.\end{aligned}$$

The last inequality follows the fact that when $P > P^{eq}$, $\sigma < P$. Substituting $\frac{\partial P^{lim}}{\partial c} > \frac{1}{\mu}\frac{1}{1-\sigma_P}$ in the previous variation of inessential patents with respect to $c$ we get:

$$\frac{d(P^{lim}-\sigma(P^{lim},c))}{dc} > 0,$$

proving that when $c$ increases, both essential and inessential patents increase in numbers.

### A.7. *Proof of Proposition 6*

Let $dP^{lim}(c,f) = \frac{\partial P^{lim}}{\partial c}dc + \frac{\partial P^{lim}}{\partial f}df$. Since by (i) and (ii) of Proposition 5 the partial derivatives are positive, there exist $dc > 0$ and $df < 0$ such that $dP^{lim}(c,f) = 0$. Now, we have:

$$\begin{aligned}d\sigma(P^{lim},c) &= \sigma_P(P^{lim},c)dP^{lim} + \frac{1}{\mu}dc \\ &= \frac{1}{\mu}dc \\ &> 0.\end{aligned}$$

Obviously, because $dp^{lim} = 0$ and the number of essential patents increases, the number of inessential patents must decrease.

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